1. Show that if a fine moduli space exists for a moduli functor, then it is unique up to isomorphism.

2. Prove that there is a fine moduli space parameterizing plane conics, and it is indeed isomorphic to the five-dimensional projective space.

3. Show that the cuspidal cubic $y^2z = x^3 \subset \mathbb{P}^2$ has a natural bijection with $\mathbb{P}^1$, but it is not a coarse moduli space for the moduli problem of lines through the origin in the plane.

4. Prove that Harder-Narasimhan filtration with respect to μ_H-stability is unique.

5. Prove that if two vector bundles E_1 and E_2 are μ_H-semistable, then $E_1 \otimes E_2$ is also μ_H-semistable.

6. Let E be a rank r vector bundle on the projective space $\mathbb{P}^n(\mathbb{C})$. Prove that if

$$H^0(\mathbb{P}^n(\mathbb{C}), (\wedge^q E)_{norm}) = 0 \;\text{ for } 1 \leq q \leq r-1,$$

then E is μ-stable.

7. Prove that there exists a rank 2 μ-stable vector bundle E on $\mathbb{P}^2(\mathbb{C})$ with $c_1(E) = -1$ and $c_2(E) = c_2$ if and only if $c_2 \geq 1$.

8. Let E be a rank 2 vector bundle on $\mathbb{P}^3(\mathbb{C})$ associated to a curve $Y \subset \mathbb{P}^3(\mathbb{C})$, i.e. we have a short exact sequence

$$0 \to \mathcal{O}_{\mathbb{P}^3(\mathbb{C})}(-c_1(E)) \to E(-c_1(E)) \to I_Y \to 0$$

Show that E is μ-stable if and only if $c_1(E) > 0$ and Y is not contained in any surface of degree $\leq \frac{c_1(E)}{2}$